Communication, Coordination, and Efficiency in Evolutionary One-Population Models *

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Abstract

We analyze the role of commitment in pre-play communication for ensuring efficient evolutionarily stable outcomes in coordination games. All players are a priori identical as they are drawn from the same population. In games where efficient outcomes can be reached by players coordinating on the same action we find commitment to be necessary to enforce efficiency. In games where efficienct outcomes only result from play of different actions, communication without commitment is most effective although efficiency can no longer be guaranteed. Only when there are many messages then inefficient outcomes are negligible as their basins of attraction become very small.

Keywords: evolutionarily stable sets, pure coordination, cheap talk.

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1 Introduction

Two-person pure coordination games typically have strict Nash equilibria that are inefficient.[1] Often it has been informally argued that communication will help players coordinate on the efficient equilibrium. However, formal game theoretic models of cheap pre-play communication also have Nash equilibria that are inefficient. For example, players may speak randomly and refuse to listen. In recent years it has been argued that those uninformative inefficient equilibria are not evolutionarily stable and that only the efficient outcomes are evolutionary stable. The reason for this result is basically twofold. First, if there are unused messages then these can be used by mutant strategies to coordinate on efficient outcomes. Second, if all messages are used because of randomization, then players must be indifferent between them and the population of strategies may drift to one where some messages are not used, which brings us back to the first case. However, this second argument is valid only when players are role-conditioned, i.e. when there is one population of player 1 types and one population of player 2 types. Examples of evolutionary stability concepts based on two-population models are Sobel's (1993) NES, Swinkels' (1992) EES Sets and Matsui's (1992) CSS. However, traditionally evolutionary models have considered symmetric games where players are drawn from a single population. In fact, many real life examples of coordination problems have this intrinsic symmetry in which individuals are identical (e.g., going through doors, shaking right or left hand, kissing once, twice or three times when meeting). Since the second argument does not hold in this case, efficiency is no longer guaranteed: mixed *Evolutionarily Stable Strategies* (ESS) that are inefficient are known to exist (Schlag, 1993, 1994, Wärneryd, 1998). These ESS are of the babbling type in that all messages are used. Of course, the efficient outcome remains an alternative prediction. The set of strategies leading to play of the efficient action is an *Evolutionarily Stable Set* (ES Set, Thomas, 1985). It is not that we find these babbling ESS particularly plausible as descriptions of real behavior. Instead, the possible emergence of inefficient outcomes and the common belief that typically the efficient outcome should result in such simple games leads us to believe that there is more behind communication than merely cheap talk.

For large message sets, we find that the babbling ESS with payoffs bounded away from the efficient are less plausible since their *invasion barrier* (i.e., the maximal size of

[1] A pure coordination game is a symmetric simultaneous move game in which each player has a finite set of pure strategies; play of the same strategy results in a strictly positive payoff, miscoordination leads to a payoff of 0.

a mutation that they can withstand) is arbitrarily small[2]. However, the mixed ESS with payoffs close to the efficient remain to have large invasion barriers.

We investigate the ability of various communication mechanisms to ensure only efficient evolutionarily stable outcomes. We choose ES Sets, a set-valued generalization of ESS, as our solution concept. Additional to cheap talk we allow for *strategy* and *hidden commitment*. Under strategy commitment, a player publicly commits to a strategy, under hidden commitment he commits to not be able to react to messages sent by others without disclosing the strategy he intends to play, e.g., he does not show up for the pre-play communication. It comes at no surprise that strategy commitment, a generalization of the endogenous timing model of Van Damme and Hurkens (1996), induces efficient outcomes. The argument relies on specific drift taking place before a mutant committing to the efficient strategy enters since simply committing to the efficient strategy must not be a best reply. Hidden commitment has the same effect of inducing efficiency through enabling beliefs to form that hidden commitment is associated to play of the efficient strategy.

Next we test the robustness of the failure of efficiency without commitment in two ways. First we assume that there are as many cheap talk messages as there are actions and that messages are associated to announcing the willingness to play an action. When the degree of commitment is exogenous, we find that inefficient outcomes persist if there is either very little commitment or almost certain commitment. Only intermediate ranges of commitment guarantee efficiency. In our second test of robustness we add external private signals. Inefficient outcomes persist without commitment opportunities whenever players can never rule out that their opponent has received the same signal.

In the second half of our paper we investigate the evolution of play under pre-play communication when players want to coordinate on choosing different actions. To our knowledge, this has not been done before. First we consider the simplest case: a *task allocation game* with two pure strategies, zero payoffs on the diagonal and two asymmetric strict equilibria on the off-diagonal. Its unique ESS is completely mixed since the strict equilibria are off-diagonal and thus cannot be reached by identical players.[3] Adding pre-play communication we find that all messages are sent in any ESS as coordination is easier when different "types" (distinguished by message sent) are matched. Players sending different (cheap talk or commitment) messages coordinate on a strict equilibrium while

[2]They only fail to exist under the implausible assumption of an infinite message set (Ayoagi, 1998).

[3]Although correct, we avoid using the term 'Battle of Sexes' that seems to imply that the game is played between players belonging to different roles and that focus rests on selection among the strict equilibria.

players sending the same cheap talk message play the (mixed) ESS of the underlying game. In particular, hidden commitment (when allowed) is associated with play of a unique action. Predictions are similar with or without commitment when there is no conflict of interest in the task allocation game, i.e., if both players always receive the same payoff. Outcomes in ESS are close to the efficient when the number of cheap talk messages is large. When there is however conflict of interest in the task allocation game then there are multiple evolutionarily stable outcomes. Inefficient ESS now exist that yield payoffs below the minimal payoff on the off-diagonal. While under pure cheap talk there is an ESS with payoff close to the efficient for large message sets, once commitment is added, evolutionarily stable outcomes are bounded away from the efficient one. In order to compare the stability of the various ESS we consider the corresponding invasion barriers. Only the nearly efficient ESS that arise under pure cheap talk have a substantial invasion barrier when there are many messages. All other ESS under pure cheap talk and all ESS in the model that allows for commitment have very small invasion barriers when there are many cheap talk messages. Thus, while adding commitment opportunities enforces efficiency in pure coordination games, it dampens evolutionary stability when players aim to choose distinct actions. Coordination on different tasks is most efficient with pure cheap talk using many messages.

Finally, we investigate pre-play communication in a symmetric 4×4 game that combines pure coordination with task allocation. There are two jobs where one is better than the other. In order to obtain a non-zero payoff, players must choose different actions within the same job. Both players always receive the same payoff. We find that strategy commitment is necessary to ensure some choices of the Good job in each evolutionarily stable outcome. Under cheap talk with or without hidden commitment there is an ESS in which only tasks within the Bad job are chosen. Once again we find that ESS with payoff bounded away from the efficient payoff have small invasion barriers for large message sets. Common interest between the players allows there to be near efficient ESS with or without commitment; their invasion barriers are never small.

2 Evolutionary Stability

In this paper we consider the following situation: Two players are drawn at random from a large (essentially, infinite) population of identical individuals to play a base game G, which is basically a coordination game. Before play starts, we allow for pre-play communication which may take the form of cheap talk or commitments. This extended game G^M is a matrix game in which players have N pure strategies $e_1, \ldots, e_N$. The $N \times N$ payoff matrix is denoted by A. Let x and y denote probability distributions on the set of pure strategies. Then the payoff of strategy x when meeting y equals $x \cdot Ay$. x is called a *Nash strategy* if $x \cdot Ax \geq y \cdot Ax$ for all strategies y.

A Nash strategy x is an *Evolutionarily Stable Strategy (ESS)* (Maynard Smith and Price, 1973) if for every strategy $y \neq x$, $y \cdot Ax = x \cdot Ax$ implies that $y \cdot Ay < x \cdot Ay$. This is equivalent to the fact that there exists an *invasion barrier* $\varepsilon > 0$ such that $y \cdot A\left(\left(1 \Leftrightarrow \varepsilon'\right) x + \varepsilon' y\right) < x \cdot A\left(\left(1 \Leftrightarrow \varepsilon'\right) x + \varepsilon' y\right)$ holds for all $y \neq x$ and $0 < \varepsilon' < \varepsilon$. In some games ESS do not exist and we therefore also consider a setwise stability concept. A subset X of the set of all Nash strategies is an *Evolutionarily Stable Set* (ES Set, Thomas, 1985) if it is nonempty and for each $x \in X$ and each y, $y \cdot Ax = x \cdot Ax$ implies that either
(i) $y \cdot Ay < x \cdot Ay$, or
(ii) $y \cdot Ay = x \cdot Ay$ and $y \in X$.
Again, this is equivalent to the fact that there exists an invasion barrier $\varepsilon > 0$ such that $y \cdot A\left(\left(1 \Leftrightarrow \varepsilon'\right) x + \varepsilon' y\right) \leq x \cdot A\left(\left(1 \Leftrightarrow \varepsilon'\right) x + \varepsilon' y\right)$ holds for all $x \in X$, y and $0 < \varepsilon' < \varepsilon$ where equality implies $y \in X$ (Balkenborg and Schlag, 1995). Any singleton ES Set contains an ESS and every ESS constitutes an ES Set as a singleton, so that ES Sets can be seen as the set-valued extension of the ESS concept.

3 Pure Coordination Games

In the base game $\mathcal{G}_1$ players have action set $K = \{1, \ldots, k\}$. They receive a_i if they coordinate on action i, and zero otherwise, in the literature often referred to as *pure coordination games*. We assume $a_1 > a_2 > \ldots > a_k > 0$. All pure strategies are evolutionarily stable and inefficient outcomes persist. We extend the game by allowing the players to send messages before they take actions. We assume that all messages are costless. We will consider two types of messages, binding and non-binding ones. A non-binding message (or cheap talk) may signal the intention to play a certain action, but the player who sent such a message is free to choose any action he likes. On the other hand, a binding message (a commitment) is a promise to play a certain action and the player cannot break his promise.

Let us start by considering cheap talk messages only. Each player will send one message m from the finite message set $M = \{m_1, \ldots, m_n\}$ with $n \geq 2$. After receiving the message of the other player he decides on his action. So a pure strategy for a player is a pair (m, f) consisting of a message $m \in M$ and a decision rule $f : M \to K$. The payoff strategy (m, f) receives when playing against (m', f') is a_i if $f(m') = f'(m) = i$ and zero otherwise. Then it is easily shown that the set of all strategies that yield the efficient payoff a_1 when played against themselves is an ES Set. In each matching only action 1 is played although it can be that the opponent would play a different action if he had received a different message. Moreover, there is no other ES Set in which the same action is played in all matchings (see Schlag, 1993, 1994). However, as pointed out by Schlag (1993, 1994) and Wärneryd (1998), cheap talk does not guarantee efficiency as there exists an ESS with payoff strictly less than a_1. Namely, consider the mixed strategy x where a player mixes uniformly over all messages and plays action k if the messages are the same and action $k-1$ otherwise. Then x is a Nash strategy that puts positive weight on each of its best replies and x earns $b = [a_k + (n-1)\, a_{k-1}]/n < a_{k-1}$ against itself. Suppose that x' is a best reply to x that uses a non-uniform probability distribution over the messages. Then x' earns less than b when meeting itself because the probability of two identical messages is more than $1/n$. Thus, x is an ESS that achieves payoffs bounded below a_{k-1} irrespective of the number of messages. Notice that in the play of x the cheap talk messages are *revealing* in the sense that each message identifies the strategy used by the player who sent this message.

We would however like to point out that the resistance to mutations (described by the size of the invasion barrier) of these inefficient ESS declines as the number of messages increases if there are more than two actions. Consider a mutant y who sends message m_1 and plays action 1 when receiving message m_1 (and the same as x when receiving message $m_i \neq m_1$). In a population in which the fraction $(1-\varepsilon)$ play x and ε play y, y performs worse than x by at most a_{k-1} when matched against an x who sent message m_1 (an event that occurs with probability $(1-\varepsilon)/n$) and better than x by a_1 when matched against y (an event that occurs with probability ε). Since the strength of x vanishes as n gets large it follows that the invasion barrier of x gets arbitrarily small once the number of messages is sufficiently large. To get a feel for the actual numbers, it is easily shown that the invasion barrier is bounded above by $1/(n+1)$.[4] On the other hand, the invasion barrier of the inefficient ESS that specifies to play action $i > 1$ when sent and received messages coincide and to play action 1 otherwise remains large. This follows from checking that $y \cdot A((1-\varepsilon)\, x + \varepsilon y) < x \cdot A((1-\varepsilon)\, x + \varepsilon y)$ for all $\varepsilon < a_i/(a_1 + a_i)$ and arguing that other

[4]Actually, a tighter upper bound is $\frac{a_{k-1}/a_1}{a_{k-1}/a_1+n}$ so when a_1, a_2, a_3 equal $10, 2, 1$ respectively, then we find that the upper bound is $\frac{2}{2+10n}$ and thus below 0.02 when there are just 10 messages.

mutants y' are even worse competitors against x.

Notice that we only showed above that the invasion barrier of the inefficient ESS that relies on playing actions a_k and a_{k-1} vanishes as the message set becomes large. Of course there are many inefficient ESS. However, our above argument is easily generalized to any sequence of invasion barriers $\varepsilon^{(n)}$ corresponding to a sequence of ESS $x^{(n)}$ for the game with n messages that satisfies $\limsup_{n\to\infty} x^{(n)} \cdot Ax^{(n)} < a_1$. Thus, given large message sets, ESS with substantial invasion barriers must yield payoffs close to the efficient.

3.1 Commitment

We will now consider the possibility of commitments. A player can pre-commit to some action i by sending message m^i. If each message is such a commitment message then we are back to the original game without cheap talk. Hence, we also allow players to send a cheap talk message from the set $\{m_1, \ldots, m_n\}$ where we now allow for $n \geq 1$. If $n = 1$ this means that players are not obliged to commit. In this case the game is exactly the game of endogenous timing considered in Van Damme and Hurkens (1996). If $n \geq 2$ then players can also "cheap talk". Again, a pure strategy for a player is a pair (m, f) as before with the restriction that $f(m') = i$ for all m' if $m = m^i$, i.e. a player that has committed to action i must play i whatever the opponent says or does. Let $M = \{m^1, \ldots, m^k\} \cup \{m_1, \ldots, m_n\}$. Now the game $\mathcal{G}_1^M$ has a unique ES Set and it earns a_1, i.e. it is fully efficient:

Theorem 1 *(Strategy Commitment) $\mathcal{G}_1^M$ has a unique ES Set, namely $X = \{x : x \cdot Ax = a_1\}$.*

Proof. Note that X is the set of mixed strategies that yield the efficient payoff against themselves. X is the set of strategies where a player either commits to action 1 or randomizes between some cheap talk messages and plays action 1 in case he receives one of the messages that were sent with positive probability. In particular, X contains the weakly dominated strategy (m_1, f) with $f(m_1) = 1$ and $f(m^i) = k$.

X is an ES Set: Suppose mutant y is a best reply against a strategy $x \in X$ but $y \notin X$. We need to show that $y \cdot Ay < x \cdot Ay$. Well, the presumptions about the mutant imply that $y \cdot Ax = x \cdot Ax$ and $y \cdot Ay < a_1$. The symmetry of the game implies that $x \cdot Ay = y \cdot Ax$. Hence, it follows that $y \cdot Ay < a_1 = x \cdot Ax = y \cdot Ax = x \cdot Ay$.

Now, suppose X' is an ES Set with $x \in X'$ and $x \cdot Ax < a_1$. If x puts positive weight on committing to action i, m^i would be a best reply (since x is a Nash strategy) that gets at most payoff a_i against x and exactly a_i against itself. The ES Set conditions imply that $m^i \in X'$, $x \cdot Ax = a_i$ and $i > 1$. But then to wait (i.e. send cheap talk messages) with probability one, and to always play i must also be in X'. Notice however that it cannot be that all individuals wait and then achieve an inefficient outcome. Whenever all individuals wait, there is no selection pressure against reactions to commitment strategies. Thus, we can assume that all individuals who wait play a best reply in case the other commits. But it is clear that m^1 is the unique best reply against such strategies. We therefore get a contradiction and, hence, no ES Set gives an inefficient payoff. □

Next we assume instead that individuals are able to publicly commit to not be able to react to messages sent by opponents. This could be done by (visibly) destroying one's receiver or taping one's ears. It can be interpreted as a credible message saying "I have already chosen my action but I do not tell you which one." This we will also refer to as *hidden commitment* as opposed to the case of *strategy commitment* considered before. Alternatively, it is as if each player is given the option of not showing up to the exchange of cheap talk, hence one could also speak of *commitment by absency*absence.. Let m^H be the message sent by an individual who commits to not react to received messages. Thus the strategy $\left(m^H, f\right)$ has the property that there exists $i \in K$ such that $f(m) = i$ for all $m \in M$. Hence we also write $\left(m^H, i\right)$. Notice that, in contrast to strategy commitment, hidden commitment does not change the information structure of the underlying game. We find that evolution leads to efficiency if players are not forced to join pre-play communication.

Theorem 2 *If $M = \{m^H\} \cup \{m_1, \ldots, m_n\}$ $(n \geq 1)$ then the unique ES Set contains only efficient payoffs.*

Proof. It follows easily that the set of strategies that yield a_1 against itself is an ES Set. In fact, this implies that any other ES Set X must contain only strategies that yield a payoff strictly below a_1 against themselves. Consider x belonging to the ES Set X and assume that $\left(m^H, i\right)$ is used with positive probability in x. Since (m^H, i) yields a_i against itself and at most a_i against any other strategy (including x), we must have that $(m^H, i) \in X$ and $i > 1$. However, then (m_1, f) with $f(m^H) = i$ and $f(m_1) = 1$ is a best response that achieves the maximal payoff against itself. This yields a contradiction. Hence, (m^H, i) with $i > 1$ is not in the support of $x \in X$. Since m^H is not sent in x, there is no evolutionary selection pressure against strategies in the support of x reacting to m^H by playing action 1. This however contradicts the fact that X is an ES Set that does not contain strategy combinations yielding the efficient payoff. □

3.2 Probabilistic Commitment

Communication without commitment is modelled above as cheap talk. Messages have no meaning (i.e., connection to later play) unless through play in an ESS. In practice however, we use words or phrases to communicate that have a meaning, such as "I plan to choose action i". Adding such a meaning does not change the results unless there is some degree of commitment or belief that the other player is committed to what he says. In the following we consider a very simple model in which messages can be attributed with a meaning. We will consider our pure coordination game from above with two actions and two messages where sending message m_i now means that the player signals that he wants to choose action i. We add an exogenous probability $\lambda \in (0,1)$ (which is the same for all players) that a player sending message m_i is committed to play action i, with probability $1 \Leftrightarrow \lambda$ sending message m_i is pure cheap talk. Whether or not the message is in fact a commitment is not observed by the opponent. Thus, a large/small λ corresponds to large/small degree of truth in what players communicate about their later intentions. The following results are easily verified by analyzing the corresponding 8×8 matrix and show that only an intermediate degree of truth (not too small and not too large) will guarantee efficiency.

The set of efficient strategies remains an ES Set for all $\lambda \in (0,1)$ (send message m_1 and play action 1 whenever receiving message m_1). For $a_1/a_2 < \frac{1-\lambda}{\lambda}$ there exists an inefficient mixed ESS with support on strategies that play action 2 when observing the same message and action 1 otherwise. For $a_1/a_2 < \frac{1}{1-\lambda}$ ($\Leftrightarrow \lambda > 1 \Leftrightarrow a_2/a_1$) there exists an ES Set with outcome a_2 (send message m_2 and play action 2 when receiving m_2). Thus, whenever a_1/a_2 is above $\max\left\{\frac{1-\lambda}{\lambda}, \frac{1}{1-\lambda}\right\}$ (which means that a_1/a_2 must be greater than $2/\left(\sqrt{5} \Leftrightarrow 1\right) \approx 1.6$) we find that evolutionary stability selects the efficient outcome a_1. In particular, when $a_1/a_2 = 2$ then this efficiency result holds for $\lambda \in \left(\frac{1}{3}, \frac{1}{2}\right)$.

Next we consider probabilistic commitment by absence.. This we model by assuming that a player does not show up to cheap talk with an exogenous probability λ and shows up to cheap talk otherwise (thus actual cheap talk occurs with probability $(1 \Leftrightarrow \lambda)^2$). In contrast to probabilistic strategy commitment, here the realization of the commitment of the opponent is observed by uncommitted players. It follows easily that there is always an inefficient ESS: when both players show up to cheap talk then each plays the same inefficient ESS of the game with only cheap talk, whenever at least one player does not participate in cheap talk then both play action k.

3.3 External Signals

Up to now we assumed that players can only condition play on messages or signals sent by other players. In the following we briefly consider the situation where players can also condition on external (or exogenous) signals. Before two players begin playing a given game (with or without pre-play communication), each of the two players receives a private signal that is not observed by the other player. For simplicity assume in the following that there are two signals s_1 and s_2. Let $p(s_i, s_j)$ be the probability that player one (two) receives signal s_i (s_j). Since players are identical, $p(s_i, s_j) = p(s_j, s_i)$. For example, an external third party flips a coin and then assigns one player to be player one and the other to be player two; hence, $p(s_i, s_i) = 0$ and $p(s_1, s_2) = 1/2$.[5] The role of the third party could also be replaced by the event of who came first to the interaction provided that ties occur with probability zero. Since the signals of the two players never coincide, this situation is called a *truly asymmetric game* (Selten, 1980). Alternatively one might imagine settings in which the order of arrival can not always be determined and hence $p(s_1, s_1) \cdot p(s_2, s_2) > 0$.

If neither of the two signals is ever received by both players simultaneously (i.e., $p(s_1, s_1) = p(s_2, s_2) = 0$), then efficiency is recovered. The set of strategies that yield the efficient payoff is the unique ES Set. The argument is as follows. Any ES Set X must contain a strategy that specifies to play a pure strategy after receiving s_1. This follows quite generally from Selten (1980).[6] Consequently, there are unsent messages after receiving signal s_1. Hence, there is no selection pressure to prevent players with s_2 signals from reacting to unsent messages by playing action 1. However, such a situation does not even constitute a Nash strategy unless s_1 recipients obtain the efficient payoff a_1. This implies that X contains a profile that yields the efficient payoff. Checking that there is a unique ES Set that contains an efficient profile, the statement is proven. Thus, evolutionary stability leads to efficiency in truly asymmetric games which is very much in the spirit of the efficiency results of Sobel (1993) for coordination games between different players.

However, if there is only a small probability that each signal can be received by both players then the above efficiency result is no longer true. Assume that both $p(s_1, s_1)$ and

[5] This signalling technology is commonly used when analyzing evolutionary stability in asymmetric games.

[6] The underlying argument is as follows. Let (y_1, y_2) be the behavioral strategy notation of an element of the ES set X and assume that e_1 is a pure strategy played with positive probability in y_1. Then (y_1, y_2) when matched against (e_1, y_2) attains a payoff of $(y_1 \cdot Ay_2 + y_2 \cdot Ae_1)/2$ which equals the payoff $(e_1 \cdot Ad_2 + y_2 \cdot Ae_1)/2$ that (e_1, y_2) attains against itself. Hence $(e_1, y_2) \in X$.

$p(s_2, s_2)$ are strictly positive. Then the set of outcomes found in an ES Set is enlarged. In particular, here external signals are of no help to eliminate inefficient evolutionarily stable outcomes. This follows from the fact that whenever X is an ES Set of a game without external signals then the set of mixed strategies that yield play of x whenever receiving either s_1 or s_2 for any $x \in X$ is an ES Set of the game with external signals. In this sense, it is evolutionarily stable for players to ignore their external signals, even if the probability of both receiving the same signal is very small.

4 2×2 Task Allocation Games

In the rest of the paper we aim to gain some understanding of play in games in which players want to coordinate on different actions. At first we consider the base game $\mathcal{G}_2$ with actions 1 and 2 and the payoff matrix

$$\begin{bmatrix} 0 & b \\ 1 & 0 \end{bmatrix}$$

where $b \geq 1$.[7]

We call $\mathcal{G}_2$ a *task allocation game* since players want to coordinate on different actions or tasks (when played among players of different types, $\mathcal{G}_2$ is more commonly known as "Battle of Sexes"). For example, two tasks need to be done and it does not matter who does which task as long as both tasks are done. $b > 1$ may indicate that task 1 is more pleasant conditional on the two players coordinating on different tasks. For example, two individuals who are walking side by side come to a door that needs to be opened and only allows for one to pass at a time. Here action 2 could describe opening the door and passing through second. Or two cars could simultaneously arrive at a four way stop and action 1 (2) could describe to drive first (second).

When there is no pre-play communication then this game has a unique ESS $\left(\frac{b}{1+b}, \frac{1}{1+b}\right)$ that yields the payoff $\frac{b}{1+b} < 1$.

4.1 Common Interest

We start with a task allocation game where both players always receive the same payoff, i.e. $b = 1$ (this makes it a *partnership game,* Hofbauer and Sigmund, 1988). Since

[7] Off-diagonal payoffs are normalized to keep notation simple.

there is a unique efficient outcome this is a game of *common interest* (Aumann and Sorin, 1989) However, since players are identical, play cannot be coordinated on this outcome. Payoffs are maximized when both actions are played with equal probability and the expected payoff is $1/2$. This strategy is the unique ESS of the game without pre-play communication. We will see below that pre-play communication will increase efficiency. However, it should be already clear now that the efficient payoff can never be reached as there will always be a positive probability that the two players matched send the same message and choose the same action and hence receive the payoff 0.

First we consider cheap talk messages only. Suppose $M = \{m_1, \ldots, m_n\}$ with $n \geq 2$.

Theorem 3 *Consider the cheap talk (partnership) game $\mathcal{G}_2^M$. Any connected ES Set is a singleton containing an ESS. The expected payoff in any ESS is $1 \Leftrightarrow \frac{1}{2n}$. ESS exist.*

Proof. There are many ESS, depending on how messages are interpreted. For example: mix uniformly over all messages. When the same message is received mix with equal probability between actions 1 and 2. When different messages are received then choose 1 (resp. 2) if the index of the sent message is lower (resp. higher) than that of the message received. With probability $1/n$ the same message will be sent and expected payoff in that case equals $1/2$. With the remaining probability $1 \Leftrightarrow 1/n$ different messages are sent in which case coordination occurs and the payoff is 1. Clearly, any best reply against the above strategy must use the same decision rule in case different messages are received. When the same message is received then both actions are best replies, but a strategy that does not mix uniformly between them will do worse against itself. Moreover, any best reply that does not use all messages with equal probability will also do worse against itself because the probability that the same message is received will then be larger than $1/n$.

Any element of an ES Set is an ESS that yields payoff $1 \Leftrightarrow 1/(2n)$. This is because elements of an ES Set are characterized in this game by three properties and hence are ESS. (i) All messages must be sent with positive probability. If some message m_i is not sent then there is no evolutionary pressure against the population reacting to the unsent message by playing action 1 which makes it a strict best response to send m_i and to play action 2. This however contradicts the fact that elements of ES Sets are Nash strategies. (ii) After receiving different messages a strict equilibrium must be played and after receiving the same message an ESS of the base game $\mathcal{G}_2$ must be played. Following Selten (1980) this is necessary to prevent mutants entering who send the same messages but react in a different way than the incumbent. (iii) Each message must be sent with equal probability in order to support a Nash equilibrium strategy with the first two properties. $\square$

In the above proof we found an ESS that contains a pure strategy in its support that always plays action 1 after pre-play communication (it sent message m_1). However, we also found that in an ESS there is no message after which players always choose such *stubborn* play. This is because each action is played equally likely among the matchings in which both players send the same message (see (ii) in the above proof). More generally, in an ESS there can be no *revealing messages* in the sense that the opponent know which pure strategy the player is using after receiving the message.

Next we add strategy commitment and assume $n \geq 1$. The basic properties of an ESS found for the case of only cheap talk extend when we add commitment. ESS exist. Each message (cheap talk or strategy commitment) will be sent with positive probability. Cheap talk senders best respond to commitment strategies, coordinate on the payoff 1 when received and sent message do not coincide and play each action with probability 1/2 when received and sent message coincide. Thus, each ESS yields the same outcome.

Adding strategy commitment possibilities will improve efficiency, but replacing a cheap talk message by a commitment possibility worsens efficiency. The reason is that an extra commitment possibility reduces the possibility of receiving the same message. On the other hand, when a cheap talk message is replaced by a commitment, the possibility of receiving the same message does not decrease, while coordination is impossible when two of the same commitment strategies are matched. Recall that with cheap talk there is still a chance of 1/2 of reaching coordination even if the same messages were sent. With n cheap talk messages and one commitment possibility any ESS yields a payoff $1 \Leftrightarrow 1/(2n+1)$, while with two commitment strategies and n cheap talk messages the payoff will be $1 \Leftrightarrow 1/(2n+2)$ (the same as with $n+1$ cheap talk messages). It is easy to check that these payoffs are necessary. In an ESS each message must be sent with positive probability and each cheap talk message must be sent equally likely. Moreover, ESS are Nash strategies. Thus it is easily verified that, given one strategy commitment possibility, the weights are $1/(2n+1)$ on the commitment message and $2/(2n+1)$ on each cheap talk message (given two strategy commitment possibilities, the weights are $1/(2n+2)$ on each strategy commitment and $1/(n+1)$ on each cheap talk message). Referring again to the properties of partnership games shows that these strategies are in fact ESS.

Finally, consider hidden commitment instead of strategy commitment. Then we find that in any ESS, hidden commitment is used with positive probability where all players using hidden commitment also choose the same action in the game. Thus, results on hidden commitment are analogous to the results when there is a single strategy commitment message. In particular, replacing hidden commitment with an extra cheap talk message improves efficiency. Given our analysis on strategy commitment, all we must show is that hidden commitment is followed by a unique action in any ESS. Assume instead that both hidden commitment possibilities are used in an ESS x. Then indifference between these two strategies implies that the population is equally likely to react to m^H by action 1 and action 2. Thus, the ESS x yields a payoff of 1/2. Cheap talk will be used in x, hence we may assume that some cheap talk senders respond to m^H by choosing action 2. Define the strategy $\tilde{x}$ as x conditional on sending cheap talk and responding to m^H by choosing action 2. Let z put equal weight on $\tilde{x}$ and on sending m^H followed by playing action 1. Then $z \cdot Ax = x \cdot Ax$ and $z \cdot Az = 1/2 + \tilde{x} \cdot A\tilde{x}/4 > 1/2 = x \cdot Az$ which contradicts the fact that x is an ESS.

4.2 Lack of Common Interest

Now let us consider the more interesting, yet more difficult case where $b > 1$. Without communication the population will obtain $b/(1+b) < 1$ in an ESS. With pre-play communication, the basic properties of an ESS found for the case where $b = 1$ carry over (see (i)-(iii) in the proof of Theorem 3). Existence is more difficult to prove as the game is no longer a partnership game. We will see that it is difficult to guarantee outcomes above 1 or even close to $\frac{1}{2}(1+b)$. When there is only cheap talk we obtain:

Theorem 4 *Let $M = \{m_1, \ldots, m_n\}$ and let $\mathcal{G}_2$ be a task allocation game with $b > 1$. Then $\mathcal{G}_2^M$ has an ESS x that yields $x \cdot Ax < 1$.*

Proof. Consider the following strategy x: Send message m_j with probability $r_j = b^{2(j-1)}/(1+b^2+b^4+\cdots+b^{2n-2})$. When the message received is the same as the one sent, play the mixed equilibrium of the base game $\left(\frac{b}{1+b}, \frac{1}{1+b}\right)$. If the message received has a higher index than the message sent, play action 2. If the message received has a lower index than the one sent, play action 1. Thus, x puts positive probability on the pure strategy e to send message m_n and then to always play action 1 and on the pure strategy $\tilde{e}$ to send message m_1 and then to always play action 2. It is easily verified that x is a Nash strategy. For a given message received, players prefer to send a message with a higher index. On the other hand, the higher the index of the own message, the higher the probability that the same messages are sent. To make players indifferent between the messages we need the probabilities r_j described above. This strategy earns (against itself) $q := x \cdot Ax = 1 \Leftrightarrow 1/(1+b+\ldots+b^{2n-1})$. It is clear that any best reply against x must use the same decision rule in case of different messages. A best reply y that differs from x only with respect to the probabilities in case of equal messages will do worse against itself: $y \cdot Ay < x \cdot Ax = x \cdot Ay$. Now consider a best reply y that uses the same pure strategies but with different probabilities than x, let us say $\beta_1, \cdots, \beta_n$. Then $y \cdot Ax = x \cdot Ax$, and $y \cdot Ay < x \cdot Ay$. To see where this inequality comes from, consider the $n \times n$ matrix B where $B_{ii} = b/(1+b)$, $B_{ij} = b$ when $i > j$ and $B_{ij} = 1$ when $i < j$. This matrix is the restriction of A to the strategies used in x. The maximization problem

$$\max_{y \in \Delta_{n-1}} y \cdot By \Leftrightarrow x \cdot By$$

has a unique solution, namely x. This follows since x satisfies the first order conditions and the function above is concave in y on Δ_{n-1}. For further details we refer to the Appendix. □

There are also ESS that give much higher payoffs for particular message sets when there is only cheap talk. Assume that there is an odd number of messages (i.e., $n = 2k+1$ for some positive integer k) and consider the strategy v that mixes with equal probability over all messages, playing the ESS of the base game in case of equal messages, playing action 2 if the index of the message received j is in the set $\{i+1, \ldots, i+k\} \pmod n$ where i is the index of the message sent and playing action 1 otherwise. The fact that v is an ESS follows from setting up the ESS conditions after verifying the following simply to prove statements. v puts positive weight on any of its best replies. $d \cdot Av = v \cdot Av$ implies $v \cdot Ad = v \cdot Av$. The corresponding payoff $v \cdot Av = \left(1 \Leftrightarrow \frac{1}{n}\right) \frac{1+b}{2} + \frac{1}{n} \frac{b}{1+b}$ is efficient in the sense that v is the unique maximizer of average payoffs among all symmetric strategy profiles of the extended game. Notice that the payoff the ESS v yields for large n approaches $(1+b)/2$.

Notice that none of the pure strategies in the support of the above efficient ESS v always plays the same action regardless of the message received. This is no coincidence according to the following result. The payoff in any ESS x that contains such stubborn pure strategies is bounded above by 1 or $2b/(1+b)$ depending on whether the fixed action is action 2 or action 1. The claim follows immediately for stubborn play of action 2 since this strategy is a best reply to x but never obtains a payoff greater than 1. The claim for stubborn play of action 1 follows from calculating the Nash strategy of the reduced game where we consider stubborn play of action 1 and combine the remaining strategies in the ESS to obtain the payoff matrix

$$\begin{bmatrix} 0 & b \\ 1 & q \end{bmatrix}$$

where q is bounded above by $(1+b)/2$.

Next we add strategy commitment. In any element of an ES Set there will always be some commitment to action 1 but never stubborn play of action 1 after sending cheap talk. We can not rule out the existence of stubborn play of action 2, however none of the cheap talk messages will be revealing. The reason that m^2 is not necessarily used relies on the fact that m^2 is only a best response to population play in which cheap talk players play action 1 against m^2 if the average payoff is below 1 which is no longer necessary.

Adding strategy commitment possibilities will not either help to guarantee an ESS payoff of at least 1. Suppose that one can commit to both actions 1 and 2 and that there are n cheap talk messages. Consider the following strategy x: with probability α commit to action 1, with probability β commit to action 2 and with the remaining probability play the ESS of the game with n cheap talk messages that yields a payoff of $q<1$ (i.e., the ESS described in Theorem 4), with the understanding that if a commitment message is received, the best reply is played, if possible. It is straightforward to check that, when $\alpha = (b^2 - bq)/(1-q+b+b^2-bq)$ and $\beta = (1-q)/(1-q+b+b^2-bq)$, x is an ESS that yields a payoff of $1-\beta \in (q,1)$. Notice that the addition of strategy commitment improves payoffs if the play among the cheap talk senders remains unchanged and $q<1$.

This follows from analyzing the reduced game with pure strategies m^1, m^2 and the ESS w of the game with only cheap talk that yields the payoff matrix

$$A' = \begin{bmatrix} 0 & b & b \\ 1 & 0 & 1 \\ 1 & b & q \end{bmatrix}. \tag{4.1}$$

It suffices to look at the reduced game since any best reply to x must use the same decision rule as x. Any best reply that uses different *relative* weights for the cheap talk messages will do worse against itself (same argument as in case of only cheap talk messages). Finally, any best reply that uses the commitment strategies with probabilities different from α and β will do worse against itself. (This is because $x^* = (\alpha, \beta, 1-\alpha-\beta)$ is an ESS in the reduced game. Proof is as before: x^* maximizes $y \cdot A'y - x^* \cdot A'y$ subject to $y \in \Delta_2$.)

Consider now the situation where the payoff q achieved among the cheap talk senders is greater than 1. Looking at the matrix A' in (4.1) we see for this case that no player will choose to commit to action 2. Here it is easily shown that there is a unique ES Set in which players mix between committing to action 1 and exchanging cheap talk. (Commit to action 1 with probability $(b-q)/(1+b-q)$). This yields the payoff $b/(1+b-q) \in (1,q)$ when played against itself. The maximal payoff attainable in an ES Set with strategy commitment and n cheap talk messages z_n is strictly below the maximal payoff attainable when there is only cheap talk. In particular, unlike pure cheap talk, z_n is bounded away from the efficient symmetric outcome $\frac{1}{2}(1+b)$ since $z_n < 2b/(1+b)$. Thus, adding strategy commitment is harmful to potential maximal payoffs but raises minimal payoffs.

To consider hidden commitment instead of strategy commitment does not affect the existence of an ESS with payoff below 1 nor the upper bound $2b/(1+b)$ on ESS payoffs. This is because, as in the case of $b=1$, hidden commitment is treated like a single strategy commitment. The proof is a simple extension of the case of $b=1$ and is thus omitted..

4.3 Large Message Sets

In the following we consider the size of the invasion barriers when the message sets are large. When there is only cheap talk then only ESS that yield payoffs close to the efficient payoff $(1+b)/2$ have an invasion barrier that does not vanish for large message sets. To see this, consider a sequence $x^{(n)}$ such that $x^{(n)}$ is an ESS when there are n cheap talk messages and $x^{(n)}$ yields a payoff below $(1+b)/2 \Leftrightarrow \delta$. W.l.o.g. assume that the weight put by $x^{(n)}$ on sending message m_i is increasing in i. Let $m=2k+1$ be such that the efficient ESS $\tilde{y}$ found in the previous section yields a payoff above $(1+b \Leftrightarrow \delta)/2$ when played against itself. For $n>m$ let $y^{(n)}$ be a mixed strategy that sends each of the first m messages with equal probability, after sending message m_i it acts like $x^{(n)}$ after receiving a message $j>m$ and behaves like $\tilde{y}$ when receiving a message $j \leq m$. Let $A^{(n)}$ denote the payoff matrix of the extended game with n cheap talk messages. Then $x^{(n)} \cdot A^{(n)} y^{(n)} \leq \frac{m}{n} b + x^{(n)} \cdot A^{(n)} x^{(n)}$ and it follows that

$$\begin{aligned}
\left(x^{(n)} \Leftrightarrow y^{(n)}\right) \cdot A^{(n)} \left((1 \Leftrightarrow \varepsilon) x^{(n)} + \varepsilon y^{(n)}\right) &\leq (1 \Leftrightarrow \varepsilon) \frac{m}{n} b + \varepsilon \left[x^{(n)} \cdot A y^{(n)} \Leftrightarrow (1+b \Leftrightarrow \delta)/2\right] \\
&\leq \frac{m}{n} b + \varepsilon \left[x^{(n)} \cdot A x^{(n)} \Leftrightarrow (1+b \Leftrightarrow \delta)/2\right] \\
&\leq \frac{m}{n} b \Leftrightarrow \varepsilon \delta/2 \ .
\end{aligned}$$

Consequently, the invasion barrier $\varepsilon^{(n)}$ of $x^{(n)}$ approaches 0 as n tends to infinity.

On the other hand the invasion barrier of the efficient ESS v based on $n=2k+1$ messages does not vanish since it is at least $1/(1+b)$ for any k. To see this, first verify that $(p \Leftrightarrow q) \cdot Ap \geq \frac{1}{b} p \cdot A(p \Leftrightarrow q) \geq 0$ holds for any q. Using the fact that $p \cdot Ap \geq q \cdot Aq$ we then obtain

$$\begin{aligned}
(p \Leftrightarrow q) \cdot A\left((1 \Leftrightarrow \varepsilon) p + \varepsilon q\right) &= (1 \Leftrightarrow \varepsilon)(p \Leftrightarrow q) \cdot Ap \Leftrightarrow \varepsilon p \cdot A(p \Leftrightarrow q) + \varepsilon(p \cdot Ap \Leftrightarrow q \cdot Aq) \\
&\geq \left(\frac{1 \Leftrightarrow \varepsilon}{b} \Leftrightarrow \varepsilon\right) p \cdot A(p \Leftrightarrow q) \geq 0
\end{aligned}$$

whenever $\varepsilon < 1/(1+b)$.

When strategy (or hidden) commitment is allowed, then as shown in Section 4.2, there will be strategy commitment in each ES Set. However, all invasion barriers approach 0 as the number of cheap talk messages n tends to infinity. The reason for this is as follows. Given our results in Section 4.2, the payoff in an ES Set is bounded above by $2b/(1+b)$. Construct the mixed strategy $\tilde{y}$ as above with $\delta = (1+b)/2 \Leftrightarrow 2b/(1+b)$. Again we obtain that the payoff difference between $x^{(n)}$ and $y^{(n)}$ is bounded above by bm/n when matched against $x^{(n)}$ and bounded above by $\Leftrightarrow\delta/2$ when matched against $y^{(n)}$. Hence, the invasion barrier must approach 0 when n tends to infinity.

4.4 External Signals and Sequential Cheap Talk

Consider now additional external signals as modelled in Section 3.3. As argued in that section, we can only expect inefficient evolutionarily stable outcomes to be eliminated if signals create a truly asymmetric contest (i.e., $p(s_1, s_1) = p(s_2, s_2) = 0$). In this case, without cheap talk we obtain two ESS and both are efficient. An ESS is induced if action 1 is played when s_1 is received and action 2 is played when s_2 is received. Similarly an ESS results if the roles of s_1 and s_2 are interchanged.

When cheap talk (and possibly strategy commitment) is included, then an ES Set exists if and only if the game is a partnership game (i.e., $b = 1$). When $b = 1$ then it is easily verified that the set of strategies that yield the efficient payoff is an ES Set. Moreover, there are no other ES Sets since the outcome 1 is the only payoff that can be attained in a pure Nash strategy. Assume now that $b > 1$. As shown in Section 3.3, any ES Set X must contain a pure strategy profile e. In this state, we can assume that an individual with signal s_1 obtains a payoff below b and does not send message m_i (in a pure strategy profile, a unique message is sent after each s_1). Then there is no selection pressure to prevent individuals with s_2 signals (who are the opponents of players with s_1 signals) from reacting to m_i by playing action 2. However, such a situation does not even constitute a Nash strategy since players with signal s_1 would strictly prefer to send m_i in order to obtain the maximal payoff b.

So pre-play communication destroys evolutionary stability whenever players are sure to be facing an opponent in a different role (Schlag, 1994, Kim and Sobel, 1995). This phenomenon will also effect evolutionary stability whenever pre-play communication occurs in several rounds before the actual game is played.

Consider the following simple model of sequential pre-play communication without external signals. There are two rounds of pre-play communication. In each round, each player can send a message m_i, $i \in \{1,..,n\}$. If the game played after communication is a task allocation game without common interest then ES Sets fail to exist. The proof follows in three easy steps. In any element of an ES Set it follows as before that all message combinations will be sent with positive probability. Conditional on the fact that the two messages sent in the first round do not coincide, the players must coordinate on the off diagonal - they agree on how to coordinate. However, in such situations each player still sends a message in round two. This will destroy any agreement on which player gets the better payoff b in the same way cheap talk destroys evolutionary stability in truly asymmetric contests. A way out of this dilemma is to only allow for communication in round two after both sent the same message in round one - continue communication only if an agreement has not been reached. In this case ESS exist. In fact, we obtain a game that is equivalent to the game with simultaneous cheap talk with n^2 messages: send m_i in round one and send m_j in round two if m_i is received in round one, $i, j \in \{1,..,n\}$.[8]

5 A Larger Task Allocation Game

Consider the following game with four pure strategies G_1, G_2, B_1 and B_2 with the corresponding payoff matrix:

$$\begin{bmatrix} 0 & b & 0 & 0 \\ b & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

where $b > 1$. This game can be seen as a composition of a task allocation game and a pure coordination game. That is, players can coordinate on different tasks in a Good job or in a Bad one. Again we consider whether evolution leads to outcomes that are close to the efficient or whether it at least causes individuals to choose only strategies within the Good job? It is clear that efficiency can not be guaranteed when there is only cheap talk: Players may use cheap talk to coordinate on the ones in the bottom right corner (as in the task allocation game analyzed in Section 4.2). All messages will be sent in equilibrium and hence there will be no extra messages to suggest play within the Good job.

Adding strategy commitment, we find that there will always be commitment to (a strategy in) the Good job but never to the Bad job. To see this, assume for example

[8]Notice that sequential cheap talk will not eliminate the inefficient ESS in pure coordination games.

that x belongs to an ES Set and that there is no strictly positive weight on commitment to G_1 in x. Then we may assume that x reacts to a commitment to G_1 by playing G_2. Consequently, committing to G_1 is a strict best reply to x since $x \cdot Ax < b$ must hold. The reason that no one will commit to the Bad job in an ES Set is as follows. Looking at the payoff matrix of the reduced form game with pure strategies $\frac{1}{2}G_1 + \frac{1}{2}G_2$, B_1 and the ESS w of the game with only cheap talk with payoff matrix

$$A' = \begin{bmatrix} b/2 & 0 & b \\ 0 & 0 & 1 \\ b & 1 & q \end{bmatrix}$$

we see that there cannot be commitment to B_1 but not to B_2 in an ES Set. When there is commitment to both B_1 and B_2, this has to be with equal probability and we can consider the reduced form game with pure strategies $\frac{1}{2}G_1 + \frac{1}{2}G_2$, $\frac{1}{2}B_1 + \frac{1}{2}B_2$ and w with payoff matrix

$$A'' = \begin{bmatrix} b/2 & 0 & b \\ 0 & 1/2 & 1 \\ b & 1 & q \end{bmatrix}.$$

For this matrix there is no ES Set that includes play of the second strategy.

However, allowing for strategy commitment does not rule out individuals choosing the Bad job: there exists an ES Set in which each strategy mixes between committing to G_1, committing to G_2 and sending cheap talk messages which are used to coordinate tasks within the Bad job in case nobody has committed himself. This follows from the fact that the reduced game with the three pure strategies G_1, G_2 and the ESS w of the game with only cheap talk that yields the payoff matrix

$$A''' = \begin{bmatrix} 0 & b & b \\ b & 0 & b \\ b & b & 1 \Leftrightarrow \frac{1}{2n} \end{bmatrix}.$$

has a unique ESS. The corresponding payoff is bounded below by $\max\left\{b/2, 1 \Leftrightarrow \frac{1}{2n+2}\right\}$, is increasing in n and approaches $b\,(2b \Leftrightarrow 1)\,/\,(3b \Leftrightarrow 2)$ for large n. The full game has no ESS because of the lack of evolutionary pressure against reaction towards commitments B_1 and B_2. However, the full set of strategies with the above properties is an ES Set.

Thus, adding strategy commitment to cheap talk substantially improves minimal payoffs in ES Sets for $b > 2$ while maximal payoffs remain approximately the same. When the cheap talk message set is large then only the invasion barriers of ES Sets with payoffs close to b (with or without commitment) remain large. In this sense, we argue that strategy commitment is very valuable, in particular for small message sets, in these games involving job coordination and task allocation. The above claim for large message sets follows from analogous arguments to ones used at the end of Section 4.3.

Notice that hidden commitment is less successful than strategy commitment in coordinating on the Good job. Under hidden commitment there exists an ESS with payoffs below 1: choose the ESS under hidden commitment from Section 4.1 for our larger task allocation game restricted to actions B_1 and B_2 (let hidden commitment be associated to choosing B_1). Then this also remains an ESS of the entire larger task allocation game.

6 Conclusion

We analyze the effect of pre-play communication on the evolutionary stable outcomes in various games involving coordination. Cheap talk alone is not sufficient to guarantee efficient outcomes when players aim to coordinate on the same action. One might say that this is due to the fact that players are forced to communicate. Adding a commitment device such as giving a player the possibility not to show up to the pre-play communication generates efficient outcomes. Communication itself need not even take place as a single cheap talk message suffices to generate efficiency. On the other hand, when players need to coordinate on choosing different actions and there is lack of common interest among the players then it is best to force pre-play communication and to rule out commitment possibilities, the more cheap talk messages the better. Players will choose to sometimes commit when given the opportunity which lowers evolutionary outcomes and even weakens stability when there are many messages. Without commitment possibilities, the most stable outcome under large message sets is when players allocate tasks fairly in that they perform each task approximately equally often. It is important that communication only takes place as long as asymmetries between players have not been developed. Further communication after an asymmetry arises will destroy evolutionary stability. In more complex games that involve coordination on different tasks within the same job we find that commitment by not showing up to pre-play communication is not enough to signal willingness to choose a task in the Good job. Allowing observable commitments to specific tasks is useful to guarantee higher evolutionary stable outcomes. Large message sets will again enforce equal (or fair) allocation of tasks within the Good job as the most stable outcome.

We chose the concept of ESS and its set-wise generalization ES Set as method for selecting outcomes for simplicity and as this is the traditional approach to evolutionary selection.[9] Alternatively one might choose to work directly with a selection dynamic such as the replicator dynamic or a more general aggregate monotone dynamic. This would tie our results to boundedly rational "optimal" learning by imitation scenarios as developed by Schlag (1998, 1999). In fact, any ESS or ES sets is under an aggregate monotone dynamic also an asymptotically stable strategy and an asymptotically stable set (in the definition where the set is attracting and each point is Lyapunov stable) respectively. Of course, generally there could be asymptotically sets that are not ES Sets. However, for the games analyzed in this paper, it can be verified (though beyond the scope of this paper) that asymptotically stable sets that are not also ES Sets fail to exist.

An alternative approach in the literature assumes that matched players belong to different populations or roles. Many interactions have this property, e.g., interactions between men and women. However, evolution under pre-play communication has drastically different predictions in these models. In this two population approach, the labelling of the strategies in the one population has no effect on the analysis. Consequently, 2×2 task allocation games with common interest are formally identical to symmetric pure coordination games. For these games, Sobel (1993) found that cheap talk and evolution always leads to efficiency if there are at least two messages (see also Schlag, 1994). With lack of common interest, evolutionarily stable outcomes of the basic game without communication are extremely unfair. Play will be at a strict equilibrium with one population always better off than the other. When pre-play communication is added, evolutionary stable outcomes cease to exist (Kim and Sobel, 1995, Schlag, 1994, see also Section 4.4). Neither commitment nor the extent of communication (i.e., number of messages) plays a role.

Other mechanisms to reach efficiency in coordination games have been suggested. The option of publicly burning money before choosing actions will not guarantee efficiency when selecting outcomes using ESS (see Ben-Porath and Dekel (1992)[10]). Sobel (1993) considered infinitely repeated games, again based on two-population models where the first few stages of the repeated games are interpreted as cheap messages (see also Balkenborg's

[9] Recently there has been lots of work with finite population dynamics. However, typically these dynamics do not select only mixed strategies which we find the natural solutions to our task allocation game.

[10] In fact, notice that after identifying L with U and D with R, that $0.75 * ODU + 0.25 * BUD$ is an ESS of the game in Fig 2.3b on page 45 in Ben-Porath and Dekel (1992).

(1995) findings on repeated games.)

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Appendix

Completion of proof of Theorem 4.

Setting up the Lagrangian and taking the derivative with respect to y_i yields

$$\lambda(b+1) = (b+1)^2 \Leftrightarrow y_i(b^2+1) \Leftrightarrow (x_1 + \ldots + x_{i-1}) \Leftrightarrow b \Leftrightarrow b^2(x_{i+1} + \ldots + x_n)$$

So the first order conditions for y_i and y_{i+1} yield

$$(y_{i+1} \Leftrightarrow y_i)(b^2+1) = \Leftrightarrow x_i + b^2 x_{i+1}$$

Using the expression for x it is easy to see that $y = x$ satisfies all first order conditions.

To see that the Lagrangian is a concave function of y, note that the bordered Hessian (the Hessian of the Lagrangian) is

$$\begin{pmatrix} 0 & \Leftrightarrow 1 & \ldots & \Leftrightarrow 1 \\ \Leftrightarrow 1 & & & \\ \vdots & & B + B^t & \\ \Leftrightarrow 1 & & & \end{pmatrix}$$

where $B + B^t$ is an $n \times n$ matrix with $2b/(b+1)$ on the diagonal and $b+1$ off the diagonal. By applying a series of operations on this matrix (first subtract the second row from row 3 through row $n+1$ to obtain zeros in all but one entry in the first column; second, subtract multiples of the first column from all other columns to obtain zeros in all but one entry in the second row; third, add the j-th column to the second column for $j = 3$ through $j = n+1$), one sees that the matrix is positive definite. $\square$